2013 California Draft Mathematics Framework

Presented by

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The slides presented here are solely the views of the presenters

Draft 0f the 2013 California Mathematics Framework can be viewed at:

http://www.cde.ca.gov/ci/ma/cf/

Objectives for this session

- Provide the overarching goal of the 2013 Draft California Mathematics Framework
- Highlight the shifts- increased focus, coherence and rigor
- Explain some of the changes between the 1997 framework and the 2013 mathematics framework
- Show the organization of the 2013 Draft California Mathematics Framework
- Point out some key points in the draft math framework
- Look at the differences between "traditional" and "integrated"
- Examine what "Math Modeling" means
- Acceleration Pathways
- Engage in a "critical area" activity on the framework

Purpose of the Standards

"The Common Core State Standards for mathematics resemble the standards of the highest-achieving nations and reflect the importance of focus, coherence and rigor. California's implementation of the CCSSM demonstrates a commitment to providing a world-class education for all students that supports narrowing the achievement gap, life-long learning, and the skills and knowledge necessary to fully participate in the 21st century global economy".

-California Draft Math Framework 2013 (Introduction Chapter, pg 3)

Purpose of California's New Mathematics Framework

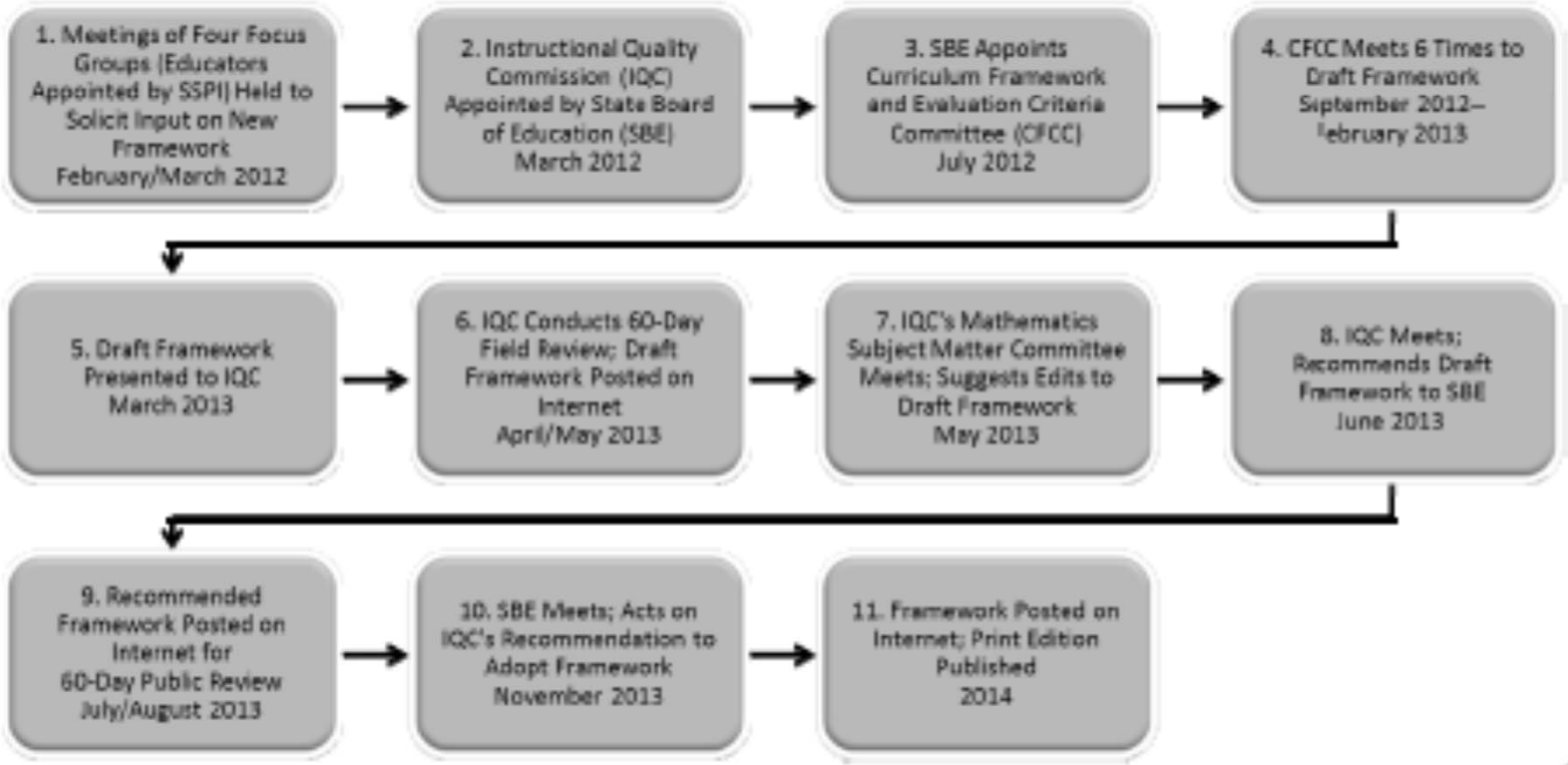
The purpose of the California Mathematics framework is to guide the curriculum development and instruction that teachers provide in their efforts to ensure that all students meet or exceed the California Common Core State Standards in Mathematics. The framework provides a context for implementing the standards in the form of guidelines for educators and developers of instructional materials. Building on the standards, the framework addresses how all students in California public schools can best meet the standards.

CALIFORNIA DEPARTMENT OF EDUCATION

Mathematics Curriculum Framework Development Process

This chart shows the major steps of the curriculum framework development process.

All meetings are open to the public.



The Common Core State Standards Requires Three Shifts in Mathematics

1. FOCUS: Focus strongly where the standards focus.
2. COHERENCE: Think across grades and link to major topics.
3. RIGOR: In major topics, pursue conceptual understanding, procedural skill and fluency, and application.

SHIFT #1: Focus Strongly Where the Standards Focus

- Narrow the scope of the content and deepen how much time and energy is spent in the classroom

- Focus deeply on what is emphasized in the standards, so that ALL students gain strong foundations

IMPLICATIONS:

- ✓Move away from "mile wide, inch deep" curricula as identified in TIMSS
- ✓Teach less, learn more!

Move from the Traditional U.S. Approach

Covering All Strands Equally in K-8

Will expand on this slide later

Number & Operations

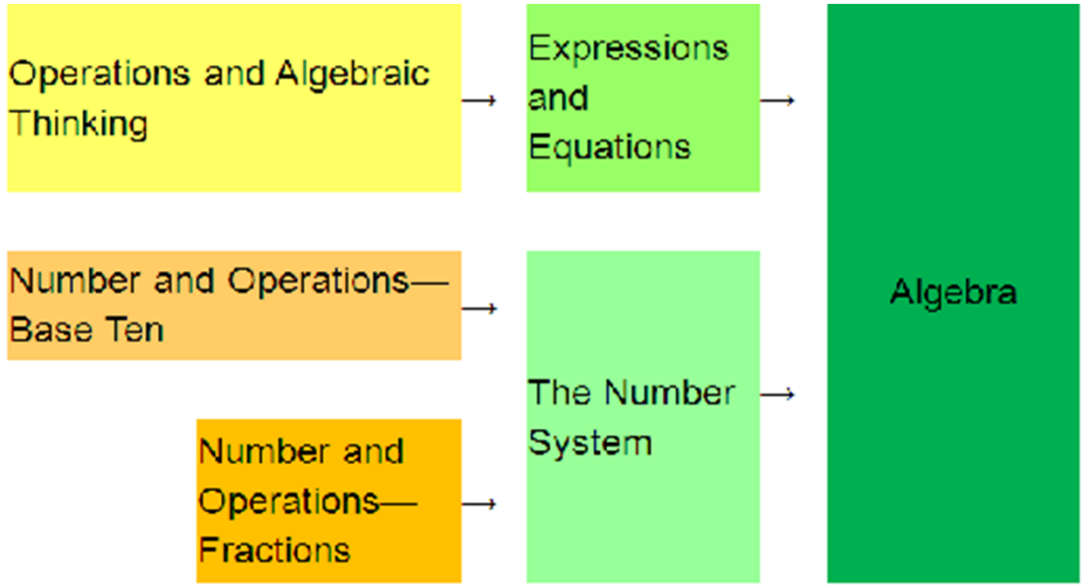
Measurement & Geometry

Algebra & Functions

Statistics & Probability

TO

Focusing on Key Concepts to get students ready for Algebra



Group Discussion

Shift #1: Focus Strongly where the Standards Focus.

In your groups, discuss ways to respond to the question, "Why focus? Why limit students to just a few concepts?

SHIFT #2: COHERENCE: Think Across Grades, and Link to Major Topics Within Grades

- Carefully connect the learning within and across grades so that students can build new understandings on foundations built in previous years.

- Build on solid conceptual understanding of core content. Each standard is NOT a new event but an extension of previous learning.

IMPLICATIONS:

✓**Do not repeat the previous year's content but build on it where applicable**

For example, students might have learned about the decimal 0.25 show them that is equivalent to 1/4 which is equivalent to 25%.

Coherence: *Think* Across Grades

Example: Fractions

"The coherence and sequential nature of mathematics dictate the foundational skills that are necessary for the learning of algebra. The most important foundational skill not presently developed appears to be proficiency with fractions (including decimals, percents, and negative fractions). The teaching of fractions must be acknowledged as critically important and improved before an increase in student achievement in algebra can be expected."

Final Report of the National Mathematics Advisory Panel

-(2008, p. 18)

Table I: Progression to Algebra I and Mathematics I in Kindergarten through Grade Eight

Kindergarten	Grade One	Grade Two	Grade Three	Grade Four	Grade Five	Grade Six	Grade Seven	Grade Eight
Know number names and the count sequence Count to tell the number of objects Compare numbers Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from Work with numbers 11-19 to gain foundations for place value	Represent and solve problems involving addition and subtraction Understand and apply properties of operations and the relationship between addition and subtraction Add and subtract within 20 Work with addition and subtraction equations Extend the counting sequence Understand place value Use place value understanding and properties of operations to add and subtract Measure lengths indirectly and by iterating length units	Represent and solve problems involving addition and subtraction Add and subtract within 20 Understand place value Use place value understanding and properties of operations to add and subtract Measure and estimate lengths in standard units Relate addition and subtraction to length	Represent and solve problems involving multiplication and division Understand properties of multiplication and the relationship between multiplication and division Multiply and divide within 100 Solve problems involving the four operations, and identify & explain patterns in arithmetic Develop understanding of fractions as numbers Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects Geometric measurement: understand concepts of area and relate area to multiplication and to addition	Use the four operations with whole numbers to solve problems Generalize place value understanding for multi-digit whole numbers Use place value understanding and properties of operations to perform multi-digit arithmetic Extend understanding of fraction equivalence and ordering Build fractions from unit fractions by applying and extending previous understandings of operations Understand decimal notation for fractions, and compare decimal fractions	Understand the place value system Perform operations with multi-digit whole numbers and decimals to hundredths Use equivalent fractions as a strategy to add and subtract fractions Apply and extend previous understandings of multiplication and division to multiply and divide fractions Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition Graph points in the coordinate plane to solve real-world and mathematical problems*	Apply and extend previous understandings of multiplication and division to divide fractions by fractions Apply and extend previous understandings of numbers to the system of rational numbers Understand ratio concepts and use ratio reasoning to solve problems Apply and extend previous understandings of arithmetic to algebraic expressions Reason about and solve one-variable equations and inequalities Represent and analyze quantitative relationships between dependent and independent variables	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers Analyze proportional relationships and use them to solve real-world and mathematical problems Use properties of operations to generate equivalent expressions Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Work with radical and integer exponents Understand the connections between proportional relationships, lines, and linear equations Analyze and solve linear equations and pairs of simultaneous linear equations Define, evaluate, and compare functions Use functions to model relationships between quantities*

Group Discussion

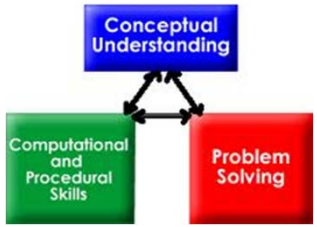
Shift #2: Coherence: Think across grades, link to major topics within grades

In your groups, discuss what coherence in the math curriculum means to you. Be sure to address both elements—coherence within the grade and coherence across grades. Cite specific examples.

Shift #3: Rigor

In Major Topics, Pursue Conceptual Understanding, Procedural Skill and Fluency, and Application

The CCSSM require a balance of:



- ✓ Solid conceptual understanding
- ✓ Procedural skill and fluency
- ✓ Application of skills in problem solving situations

Pursuit of all three requires equal intensity in time, activities, and resources.

Solid Conceptual Understanding

- Allow students to see math as more than a set of mnemonics or discrete procedures.
- Support students' ability to access concepts from a number of perspectives than teaching more of "how to get the answer".
- Conceptual understanding supports the other aspects of rigor (fluency and application)

Fluency

- The standards require speed and accuracy in calculation.
- Teachers structure class time and/or homework time for students to practice core functions such as single-digit multiplication so that they are more able to understand and manipulate more complex concepts

Required Fluencies in K–6

Grade	Standard	Required Fluency
K	K.OA.5	Add/subtract within 5
1	1.OA.6	Add/subtract within 10
2	2.OA.2 2.NBT.5	Add/subtract within 20 (know single-digit sums from memory) Add/subtract within 100
3	3.OA.7 3.NBT.2	Multiply/divide within 100 (know single-digit products from memory) Add/subtract within 1000
4	4.NBT.4	Add/subtract within 1,000,000
5	5.NBT.5	Multi-digit multiplication
6	6.NS.2,3	Multi-digit division Multi-digit decimal operations

California Common Core State Standards

2 types of standards

Mathematical Content Standards

K-8th Content Standards

- Kinder
- 1st Grade
- 2nd Grade
- 3rd Grade
- 4th Grade
- 5th Grade
- 6th Grade
- 7th Grade
- 8th Grade
- Glossary

High School Content Standards

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics & Probability
- Glossary

Appendix A

Different at each grade level

Mathematical Practice Standards (K-12)

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Recurring practices Grades K-12

"Practice" in school mathematics

1. School mathematics has always been about "practice," but it has been more about *drill and practice*, not *the practice of mathematics*.
2. The Common Core State Standards offer the possibility of re-orienting school mathematics around a more robust conception of mathematical competence.
3. The mathematical practice standards support the advancement of equity in mathematics instruction.

More on math practices in a different session!

Chapter Organization of Draft Mathematics Framework

- Introduction
- Overview of Grade Level and Course-Level Chapters
  - Grade Level: ◆Transitional Kindergarten, ◆Kindergarten, ◆Grade 1, ◆Grade2, ◆Grade 3, ◆Grade 4, ◆Grade 5, ◆Grade 6, ◆Grade 7, ◆Grade 8
  - Course: ◆Algebra I, ◆Geometry, ◆Algebra II, ◆Mathematics I, ◆Mathematics II ◆Mathematics III, ◆Pre-Calculus, ◆Prob. & Stats
- Universal Access
- Instructional Strategies
- Supporting High Quality Common CoreMathematics Instruction
- Technology
- Criteria for Evaluating Instructional Materials
- Appendices: ◆Acceleration Options, ◆List of Adaptions for Students With Learning Disabilities, ◆Modeling, ◆Financial Literacy
- Glossary

Adding California Standards to the K-12 Common Core

- In K-8 California enhanced one standard in Grade 2: MD 7
- In High School, California enhanced one standard and added 11 standards

Algebra I	Math I
A-REI 3.1 F-LE6	A-REI 3.1
Geometry	Math II
G-SRT8.1 G-C5(enhanced) G-GMD5 G-GMD6	F-LE6 G-C5(enhanced) G-SRT8.1 G-GMD5 G-GMD6
Algebra II	Math III
A-RE3.1 F-LE4.1 F-LE4.2 F-LE4.3 F-TF2.1 G-GPE3.1	F-LE4.1 F-LE4.2 F-LE4.3 F-TF2.1 F-TF2.1 G-GPE3.1

Possible Content Overlap Between CA-97 and CCSS

Note: The California Framework does not talk about the overlaps!

Grade	Mathematics
K	53%
1	82%
2	72%
3	68%
4	74%
5	67%
6	67%
7	50%
8	--

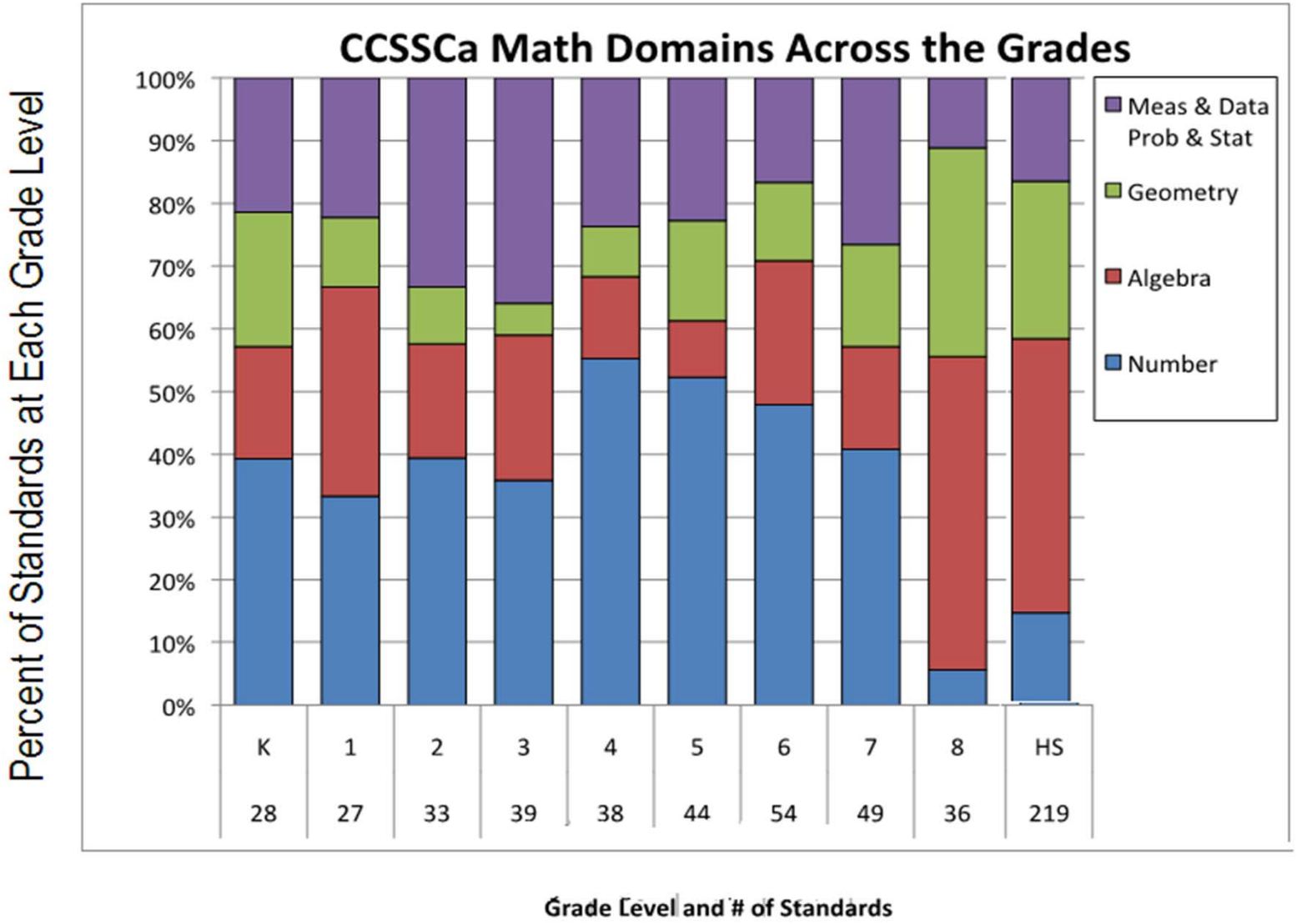
Taken from Michelle Lenertz's presentation, Coordinator, SBCSS

Examples of Grade Level Shifts

Concept	1997 Standard	CCSS
Compose simple shapes to form larger shapes (e.g., 2 triangles to form a rectangle)	Grade 2	Kindergarten
Introduction of fractions as numbers	Grade 2	Grade 3
Add and subtract simple fractions	Grade 3	Grade 4
Introduction of Integers	Grade 4	Grade 6
Dividing fractions by fractions	Grade 5	Grade 6

adapted from © 2001 California County Superintendents Educational Services Associations, Mathematics General

Standards Across the Grades



Taken from Michelle Lenertz's presentation, Coordinator, SBCSS

Example of Specificity in 2013 Standards

1997 CA Mathematics Framework

3.1 Compare fractions represented by drawings or concrete materials to show equivalency and to add and subtract simple fractions in context (e.g., ½ of a pizza is the same amount as ²⁄₄ of another pizza that is the same size; show that ³⁄₈ is larger than ¼).

2013 CA Draft Mathematics Framework

4NF.1 Explain why a fraction a/b is equivalent to the fraction (n x a)/ (n x b) by using visual fraction models, with attention to how the number and size of the parts differ even though the fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

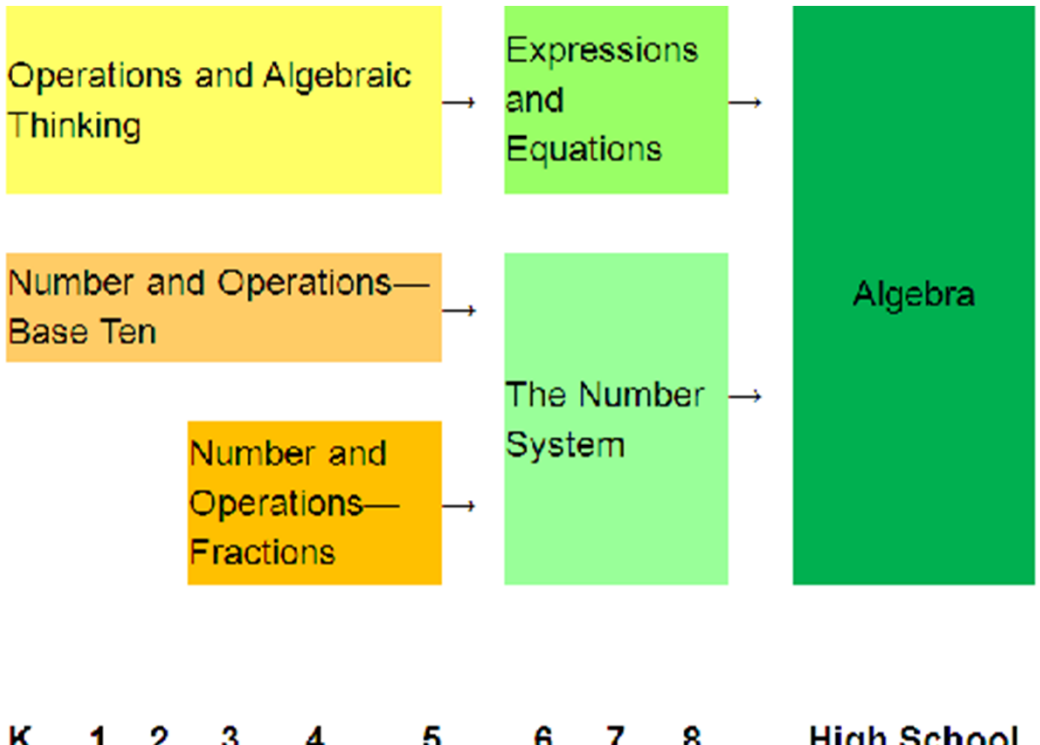
K-8 Mathematics Domain Progressions

Domains	K	1	2	3	4	5	6	7	8
Counting and Cardinality									
Operations and Algebraic Thinking									
Number and Operations in Base Ten									
Number and Operations - Fractions									
Ratios and Proportional Relationships									
The Number System									
Expressions and Equations									
Functions									
Measurement and Data									
Geometry									
Statistics and Probability									

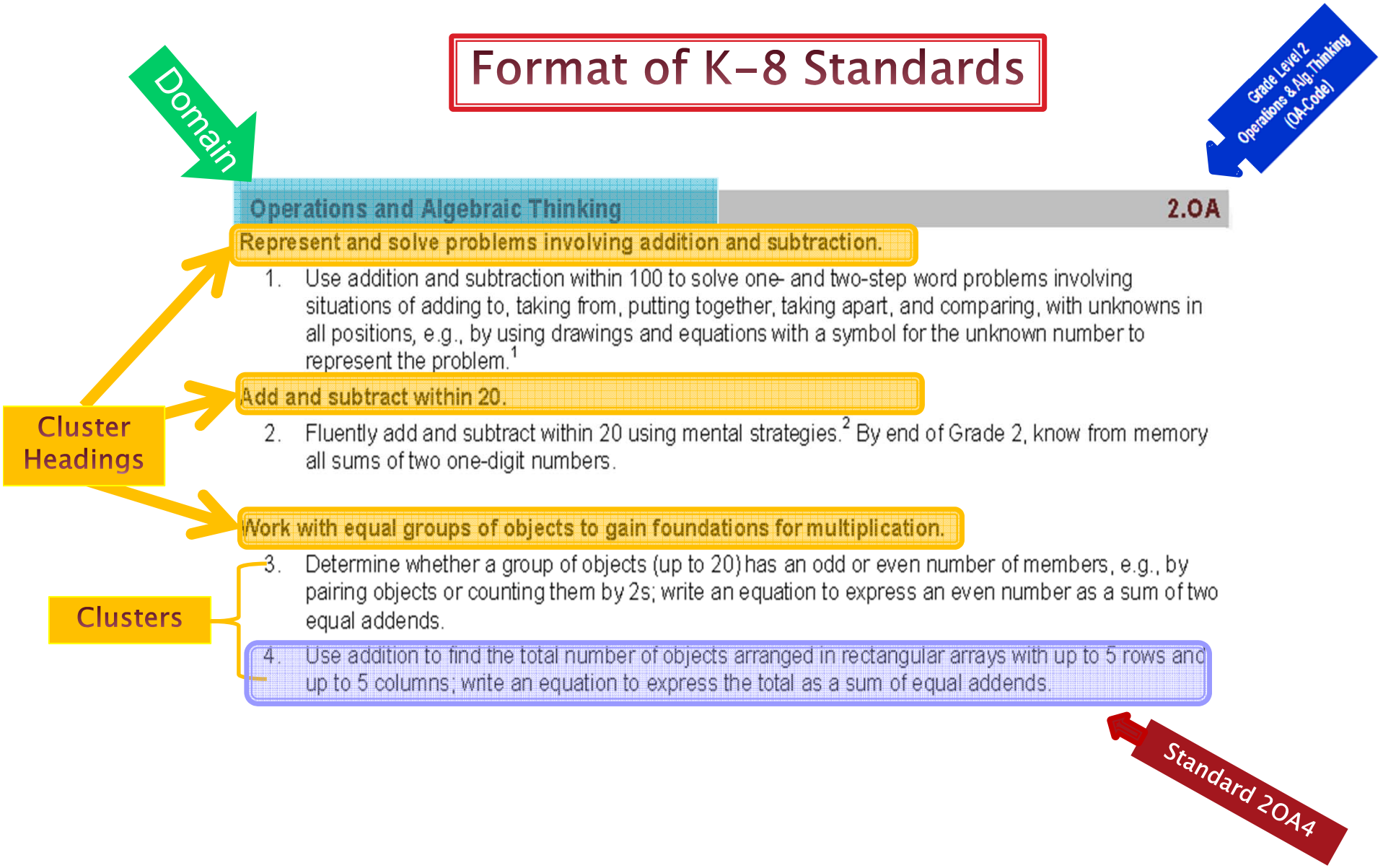
When you examine this grade level progression, what does it tell you?

K-8 Standards Progressions Provides A Strong Foundation for Algebra

- Focus on place value, operations, and fractions in early grades.
- Increase attention to proportionality and probability and statistics in middle grades.
- In depth study of linearity and introduction of functions in Grade 8.



Format of K-8 Standards



Grade Level Introduction

WHAT STUDENTS LEARN IN GRADE TWO

In grade two, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes. Students also work towards fluency with addition and subtraction within 100. By the end of grade two, students know from memory all sums of two one-digit numbers.

(1) Students extend their understanding of the base-ten system. This includes ideas of counting by fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

(2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction; and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

(3) Students recognize the need for standard units of measure (centimeter and inch), and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

4) Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Grade 2- Cluster –Level Emphasis

Operations and Algebraic Thinking
[m]: Represent and solve problems involving addition and subtraction. (2.OA.1▲)
[m]: Add and subtract within 20. (2.OA.2▲)
[a/s]: Work with equal groups of objects to gain foundations for multiplication. (2.OA.3-4)

Number and Operations in Base Ten
[m]: Understand place value. (2.NBT.1-4▲)
[m]: Use place value understanding and properties of operations to add and subtract. (2.NBT.5-9▲)

Measurement and Data
[m]: Measure and estimate lengths in standard units. (2.MD.1-4▲)
[m]: Relate addition and subtraction to length. (2.MD.5-6▲)
[a/s]: Work with time and money. (2.MD.7-8)
[a/s]: Represent and interpret data. (2.MD.9-10)

Geometry
[a/s]: Reason with shapes and their attributes. (2.G.1-3)

Explanations of Major, Additional and Supporting Cluster-Level Emphases

Major [m] (▲) clusters – areas of intensive focus where students need fluent understanding and application of the core concepts. These clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness.

Supporting [s] clusters – rethinking and linking; areas where some material is being covered, but in a way that applies core understanding; designed to support and strengthen areas of major emphasis.

Additional [a] clusters – expose students to other subjects; may not connect tightly or explicitly to the major work of the grade. *A Note of Caution: Neglecting material will leave gaps in students' skills and understanding and will leave students unprepared for the challenges of a later grade.

Connecting Mathematical Practices and Content

MP Standards	Explanation and Examples
MP.1. Make sense of problems and persevere in solving them.	In second grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. They ma use concrete objects or pictures to help them conceptualize and solve problem They may check their thinking by asking themselves, "Does this make sense?" They make conjectures about the solution and plan out a problem-solving approach. An example for this might be giving a student an equation and havin him/her write a story to match.
MP.2. Reason abstractly and quantitatively.	Younger students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities. Second graders begin to know and use different properties of operations and relate addition and subtraction to length. In second grade students represent situations by decontextualizing tasks into numbers and symbols. For example, in the task, "There are 25 children in the cafeteria, and they are joined by 17 more children. How many students are in the cafeteria? " Students translate the situation into an equation, such as: 25 + 17 = __ and then solve the problem. Students also contextualize situations during the problem solving process. For example, while solving the task above students might refer to the context of the task to determine that they need to subtract 19 if 19 children leave.
MP.3. Construct viable arguments and critique the reasoning of others.	Second graders may construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They practice their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?", "Explain your thinking," and "Why is that true?" They not only explain their own thinking, but listen to others' explanations. They decide if the explanations make sense and ask appropriate questions. Students critique the strategies and reasoning of their classmates. For exampl to solve 74 - 18, students may use a variety of strategies, and after working on the task, they might discuss and critique each others' reasoning and strategies citing similarities and differences between various problem-solving approaches
MP.4. Model with mathematics.	In early grades, students experiment with representing problem situations in multiple ways including writing numbers, using words (mathematical language) drawing pictures, using objects, acting out, making a chart or list, creatin equations. Students need opportunities to connect the different representations and explain the connections. They should be able to us any of these representations as needed. In grade two students model real-life mathematical situations with a number sentence or an equation and check to make sure that their equation accurately matches the problem context. They use concrete manipulatives and pictorial representations to explain the equation. They create an appropriate problem situation from an equation. For example, students create a story problem for th equation 43 + 17 = ___ such as "There were 43 gumballs in the machine. Tom poured in 17 more gumballs. How many gumballs are now in the machine?"

MP Standards	Explanation and Examples
MP.5. Use appropriate tools strategically.	In second grade, students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be better suited than others. For instance, second graders may decide to solve a problem by drawing a picture rather than writing an equation. Students may use tools such as snap cubes, place value (base ten) blocks, hundreds number boards, number lines, rulers, virtual manipulatives, and concrete geometric shapes (e.g., pattern blocks, three-dimensional solids). Students understand which tools are the most appropriate to use. For example, while measuring the length of the hallway, students can explain why a yardstick is more appropriate to use than a ruler.
MP.6. Attend to precision.	As children begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and when they explain their own reasoning Second grade students communicate clearly, using grade-level appropriate vocabulary accurately and precise explanations and reasoning to explain their process and solutions. For example, while measuring an object, students carefully line up the tool correctly to get an accurate measurement. During tasks involving number sense, students consider if their answer is reasonable and check their work to ensure the accuracy of solutions.
MP.7. Look for and make use of structure.	Second graders look for patterns. For instance, they adopt mental math strategies based on patterns (making ten, fact families, doubles). Second grade students look for patterns and structures in the number system. For example, students notice number patterns within the tens place as they connect skip counting by 10s to corresponding numbers on a 100s chart. Students see structure in the base-ten number system as they understand that 10 ones equal a ten, and 10 tens equal a hundred. Students adopt mental math strategies based on patterns (making ten, fact families, doubles). They use structure to understand subtraction as a missing addend problems (e.g., 50 - 33 = __ can be written as 33 + __ = 50 and can be thought of as "How much more do I need to add to 33 to get to 50?")
MP.8. Look for and express regularity in repeated reasoning.	Second grade students notice repetitive actions in counting and computation (e.g., number patterns to skip count) When children have multiple opportunities to add and subtract, they look for shortcuts, such as using estimation strategies and then adjust the answer to compensate. Students continually check for the reasonableness of their solutions during and after completing a task by asking themselves, "Does this make sense?"

Grade Level 2 Overview

Grade 2 Overview

Operations and Algebraic Thinking

- Represent and solve problems involving addition and subtraction.
- Add and subtract within 20.
- Work with equal groups of objects to gain foundations for multiplication.

Number and Operations in Base Ten

- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data

- Measure and estimate lengths in standard units.
- Relate addition and subtraction to length.
- Work with time and money.
- Represent and interpret data.

Geometry

- Reason with shapes and their attributes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

High School Organization: Conceptual Categories, grades 9–12

- Number and Quantity (N)
- Algebra (A)
- Functions (F)
- Geometry (G)
- Modeling (*)
- Statistics and Probability (S)

High School Standards

- **Conceptual Categories**
  - Across course boundaries
  - Span high school years

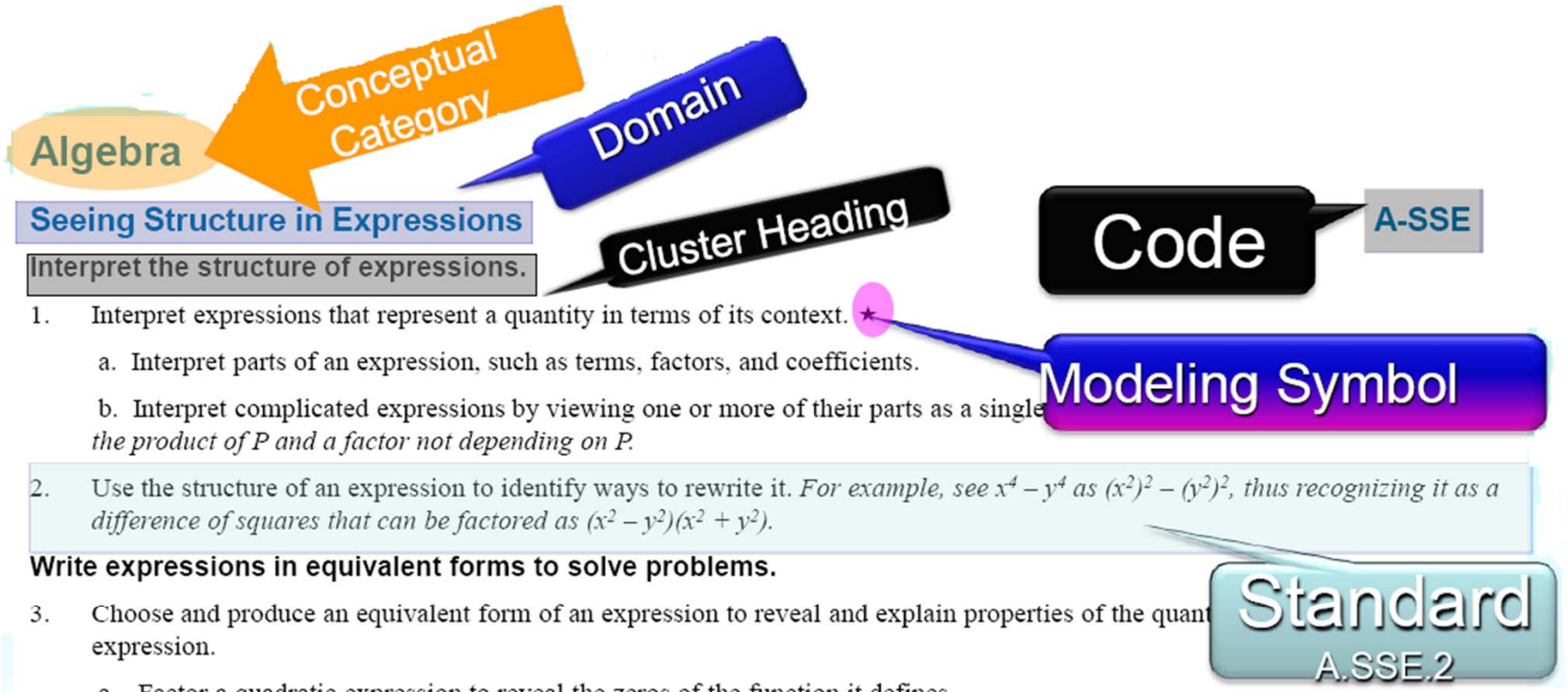
- **Standards**

– "Core" for common mathematics curriculum for ALL students to be college and career ready

– "College Ready" for entry level credit courses

(+) Additional courses that students should learn in order to take courses such as pre-calculus, calculus, Advanced Placement Calculus and Advanced Placement Statistics

Format of High School Standards



Format of a Higher Level Course- Pre-Calculus

Number and Quantity — Conceptual Category

The Complex Number System — Domain

N –CN — Code

Perform arithmetic operations with complex numbers. — Cluster Heading

3. (+)Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\,i)^3 = 8$ *because* $(-1 + \sqrt{3}\,i)$ *has modulus* 2 *and argument* 120°. — Standard N –CN5
6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

High Level Standard

What isn't mathematical modeling?

The terms "model" and "modeling" have several connotations, and while the term "model" has a general definition of "using one thing to represent something else," *mathematical modeling* is something more specific. Below is a list of some things that are *not* mathematical modeling in the sense of the CCSSM.

- It is not modeling in the sense of, "I do; now you do."
- It is not modeling in the sense of using manipulatives to represent mathematical concepts (these might be called "using concrete representations" instead.)
- It is not modeling in the sense of a "model" being just a graph, equation, or function. Modeling is a process.
- It is not just starting with a real world situation and solving a math problem; it is returning to the real world situation and using the mathematics to inform our understanding of the world. (I.e. contextualizing and de-contextualizing, see MP.2.)
- It is not beginning with the mathematics and then moving to the real world; it is starting with the real world (concrete) and representing it with mathematics.

What is mathematical modeling?

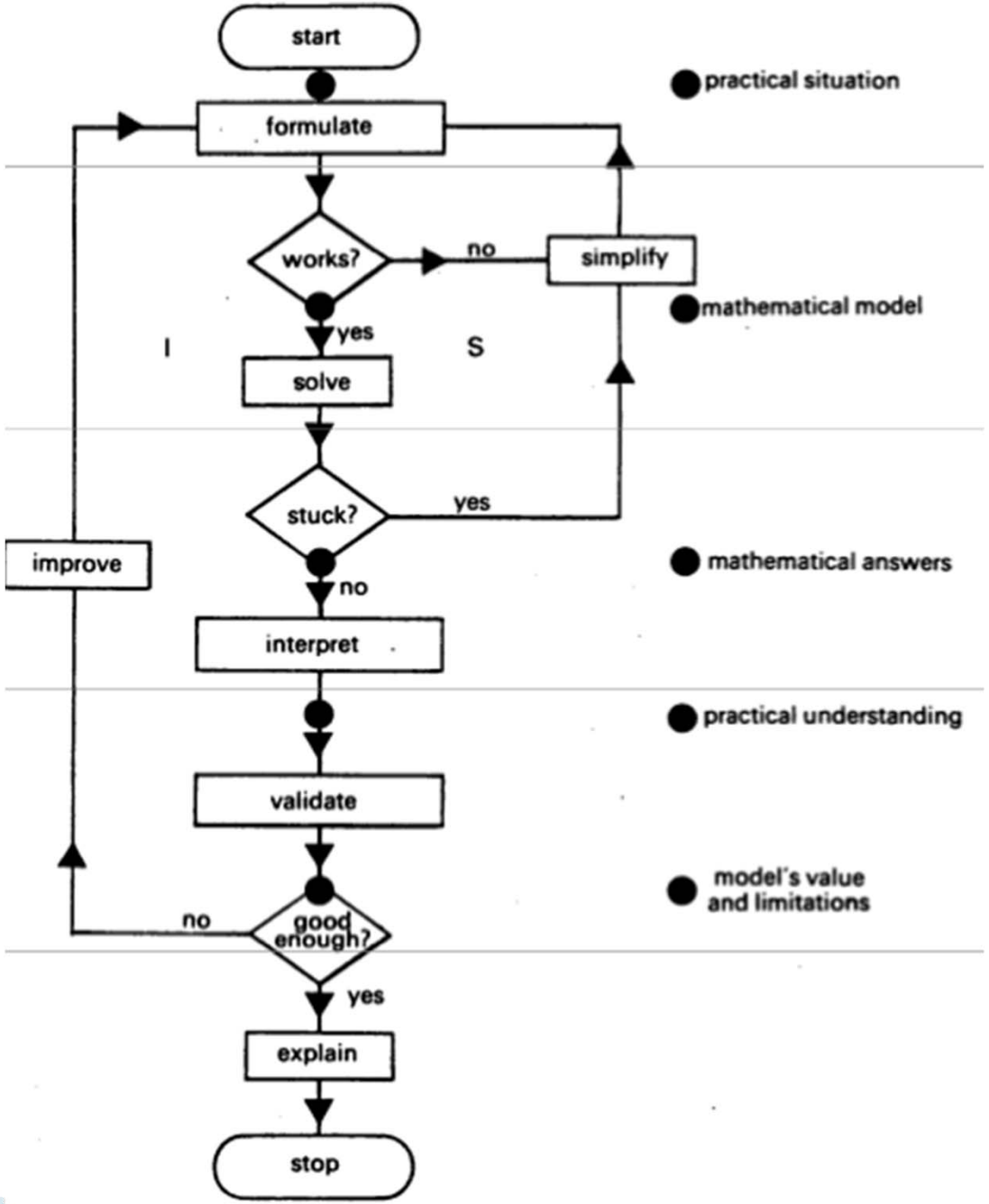
According to Joshua Paul Abrams, a teacher of mathematical modeling:

"**Mathematical modeling** is the **process** of using mathematics to study a question from outside the field of mathematics. A **mathematical model** is a **representation** of a particular phenomenon using structures such as graphs, equations or algorithms."

Joshua Paul Abrams, Mathematical Modeling, 2001. p.2.

"They [students] should spend considerable time using math to resolve problems for which mathematics is neither the initial source of their motivation nor the end result. When we teach mathematics as an intellectual lens through which non-mathematical questions can be examined, then we are providing our students with an education that will serve them and our society throughout their lives."

Joshua Paul Abrams, Mathematical Modeling, 2001, p.4.


start
formulate
works?
no
simplify
yes
I
S
solve
stuck?
yes
improve
no
interpret
validate
no
good enough?
yes
explain
stop
practical situation
mathematical model
mathematical answers
practical understanding
model's value and limitations

Modeling In a Nutshell

Start With Life

- Problem to solve, question to answer, decision to make, real-life situation to understand.
- Life can be complex → Simplify!

Do The Math!

- Use available tools, models, representations.
- Learn new ones (and the requisite mathematics)
- Attend to accuracy, appropriateness, relevance

Check Back With Life

- Apply, revise, extend.
- Cycle through the process as many times as needed.

Bruce Grip, 2012

Our Challenge As Teachers

Modeling • Teaching Math • Teaching Modeling

Mathematics Teacher and Curriculum Writer

- Begins with mathematics (content standards).
- Focus on mathematics; uses applications to illustrate, motivate, develop or understand mathematics.
- Expected outcome is deeper understanding of the math

Modeler and Modeling

- Begins with life. (Math provides the tools and pathways.)
- Focus on understanding or solving a life problem or improving a product or situation.
- Leads to a solution, decision, recommendation, modification, plan

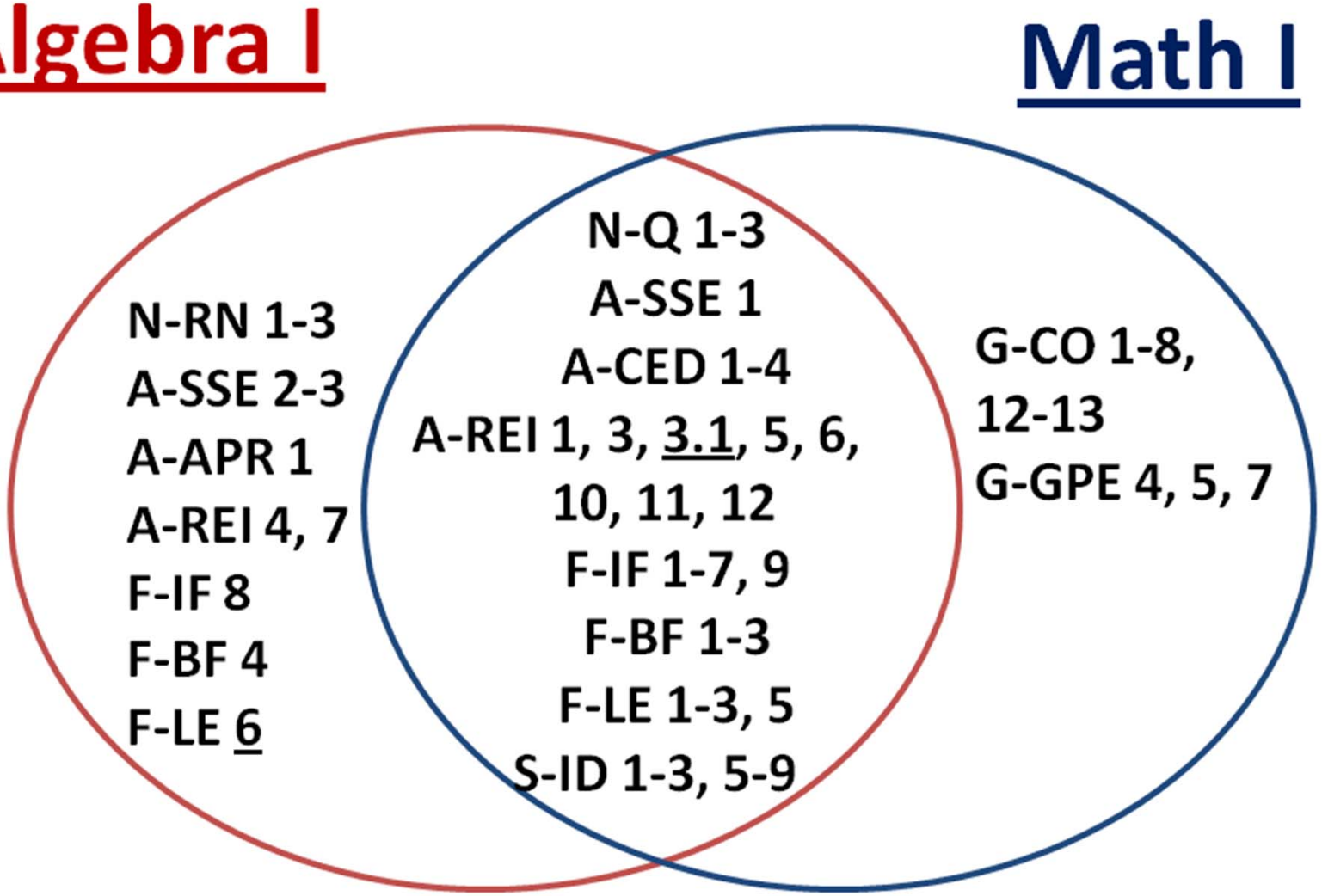
Bruce Grip, 2012

A course in Mathematical Modeling should:

- Be in addition to, not a replacement for, the incorporation of mathematical modeling into the fabric of all Higher Mathematics courses. The conceptual category of Modeling was not intended as a separate course that students may or may not encounter in high school.
- Deepen a student's understanding of, and experience with, all stages of the mathematical modeling process. The course should be about modeling as well as mathematics, and the relationship between the two.
- Allow for sufficient opportunities for students to apply mathematical content already learned to unique problems and contexts.
- Challenge and motivate students to recognize the **need** to learn, and then apply, new mathematics and related models. Once the models and the mathematics are introduced, students are challenged to find other contexts to apply the same or a similar model.
- Progressively allow students more and more freedom and opportunities to formulate their own questions, develop and apply and justify their own mathematical models, and analyze and defend their own conclusions through collaboration and dialog with peers and teachers. (This will be a challenge for teachers to provide the right balance of freedom and support. Students need to struggle for learning to take place but not become so discouraged they quit. The teacher will need to know which scaffolds to use and will need to develop open-ended questions to support and sometimes guide students thinking.)
- Help students and teachers recognize that mathematical modeling is something we all do to some extent every day, by developing two related dispositions: (1) the ability to look at a life situation and wonder how mathematics might be applied to understand or solve the situation and, (2) the ability to look at a mathematical concept and wonder how it might be applied to life experiences.
- Provide opportunities for students to tackle real-world problems of varying complexity, from things as ordinary as figuring out which coupon to use, which phone plan to choose, or how much tip to leave, to things as as complex as how to best prepare for a disaster, how to solve a crime, how to rate products, or how to balance the need for increased energy supplies with the need to protect the environment.
- Allow for the learning of mathematical principles, "big ideas," concepts, procedures, standard models, and skills in a meaningful setting, to establish meaning and relevance before teaching mathematics whenever possible.

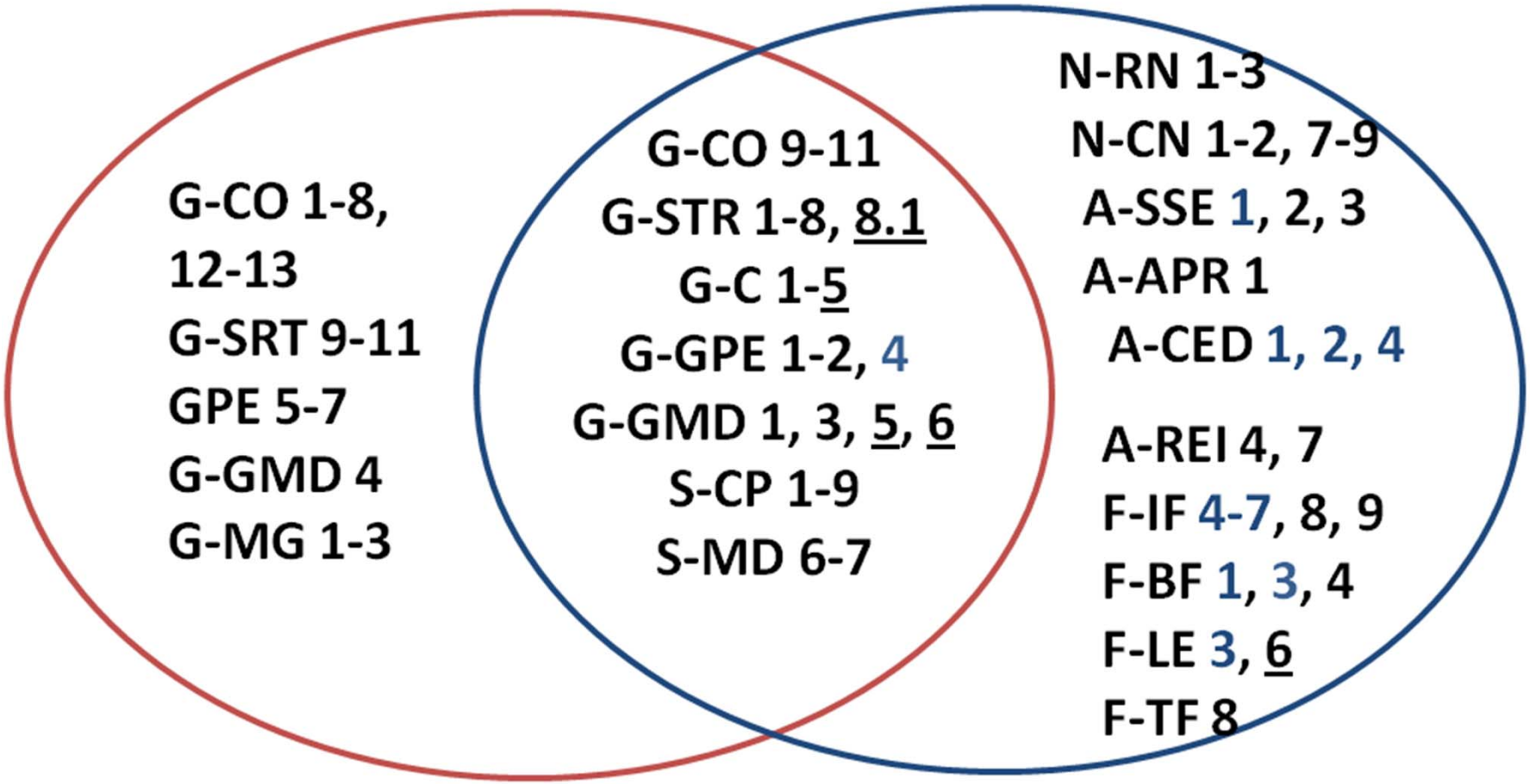
Comparison of the Common Core Mathematics Courses to the 1997 California Mathematics Courses

Common Core Grade 8	▪ Pre-Algebra ▪ Some "1997 Geometry" ▪ "1997 Algebra 1A"	**Common Core Algebra 1**	▪ "1997 Algebra 1B" ▪ "1997 Algebra 2A" ▪ Some Statistics Topics
Common Core Geometry	▪ More "1997 High School Geometry" ▪ Transformational Geometry ▪ "1997 Algebra 2" Probability	**Common Core Algebra 2**	▪ "1997 Algebra 2B" ▪ Introductory Trig ▪ (sine, cosine) ▪ Some AP Statistics



Underlined standard is California revised addition

From the work of Sunny Chin- Look
- CFCC Member and National Board Certified Teacher



Underlined standards are California revised addition.
Standards in blue are also in Math I.

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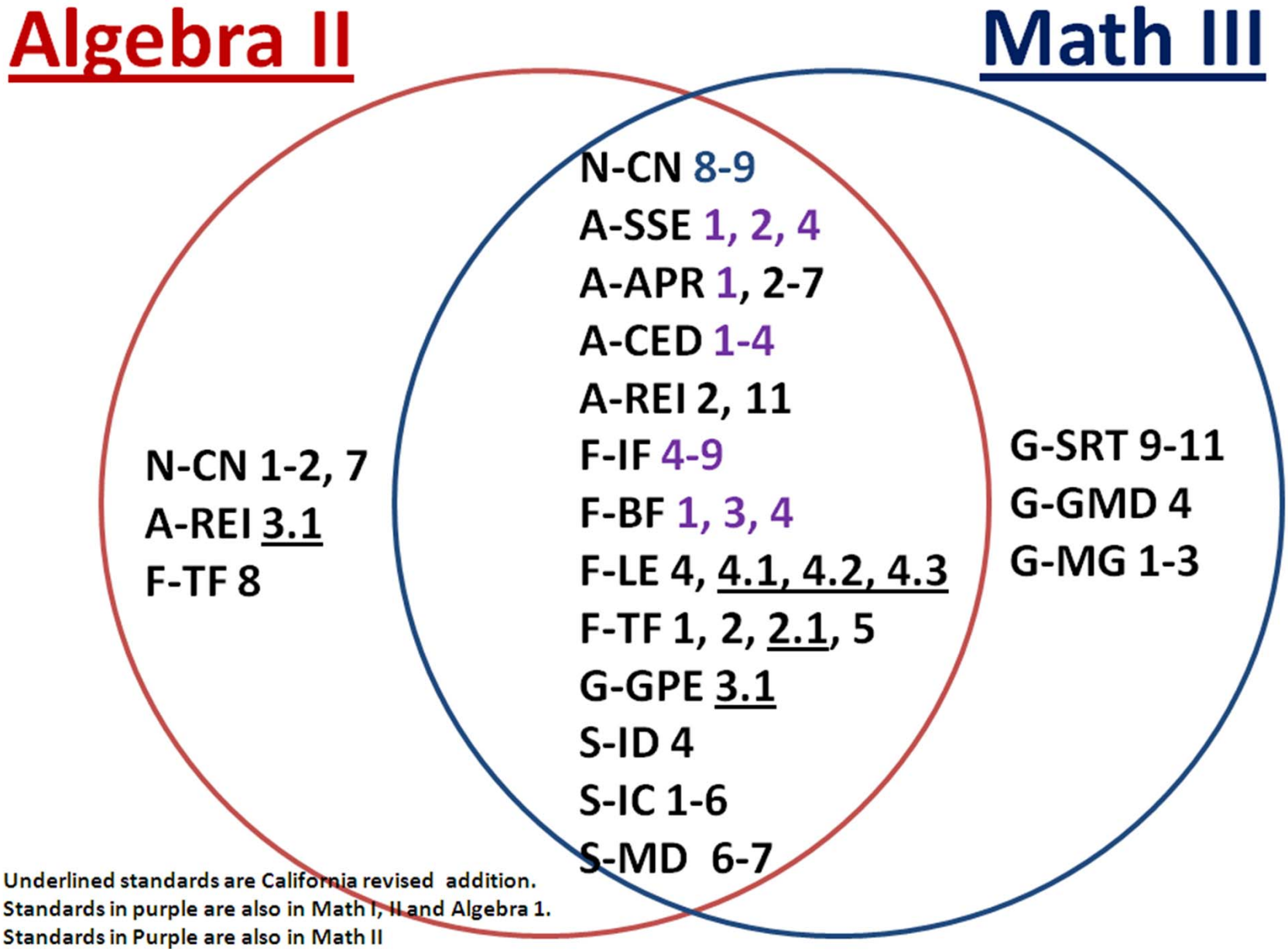

Underlined standards are California revised addition.
Standards in purple are also in Math I, II and Algebra 1.
Standards in Purple are also in Math II

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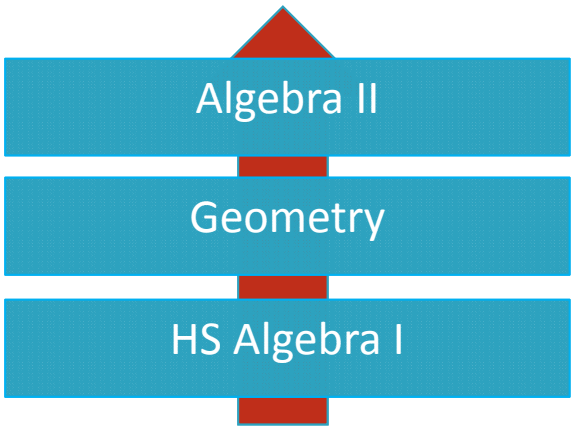
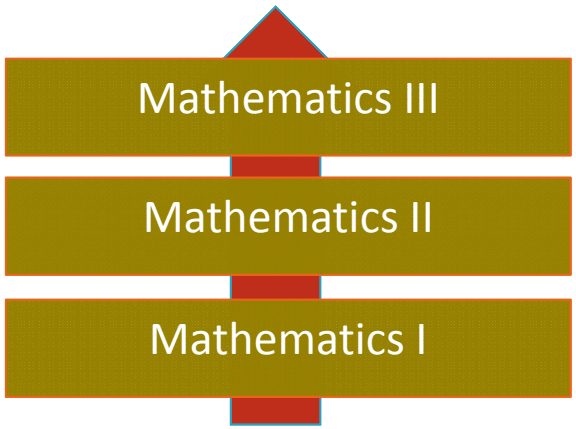
High School

Two Mathematics Pathways

Courses in higher level mathematics:
Pre-calculus, AP Calculus, AP Statistics, or courses designed for career technical programs of study.



TRADITIONAL Pathway
(Typical *in* U.S.)
2 Algebra courses, 1 Geometry course, with Probability and Statistics interwoven



INTEGRATED Pathway
(Typical *outside* of U.S.)
3 courses that attend to Algebra, Geometry, and Probability and Statistics each year

adapted from 2011 © CA County Superintendents Educational Services Association

Traditional Pathway

MS Domains	7th Grade CC	8th Grade CC	HS Domains	Model Algebra 1 (Focus on linear, quadratic, exponential, & absolute value)	Model Geometry	Model Algebra 2
Number System:	• Arithmetic operations on rational numbers	• Know that there are irrational numbers & approximate them by rationals.	**Number & Quantity:**	• Extend properties of exponents to rational exponents • Use properties of rational & irrational numbers • Reason quantitatively & use units to solve problems		• Perform arithmetic operations with complex numbers • Use complex numbers in polynomial identities & equations
Expressions & Equations (including Ratio & Proportions):	• Unit rates • Recognize & represent proportional relationships • Solve multistep ratio & percent problems • Generate equivalent expressions • Solve problems using numerical/algebraic expressions & equations	• Know and apply properties of exponents • Use square root & cube root to solve equations • Use scientific notation to estimate and compare quantities • Perform operations with numbers expressed in scientific notation • Graph proportional relationships interpreting unit rate as slope. Compare different proportional relationships represented in different ways. • Use similar triangles to explain why slope is a constant between any two distinct points on a line on the coordinate plane • Derive and use equation $y = mx + b$ • Solve linear equations in one variable • Analyze & solve linear systems	**Algebra:**	• Interpret the structure of expressions • Write expressions in equivalent forms to solve problems • Perform arithmetic operations on polynomials • Create equations that describe numbers or relationships • Understand solving equations as a process of reasoning & explain the reasoning • Solve equations & inequalities in one variable • Solve systems of equations • Represent & solve equations and inequalities graphically		• Interpret the structure of expressions • Write expressions in equivalent forms to solve problems • Perform arithmetic operations on polynomials • Understand the relationship between zeros & factors of polynomials • Use polynomial identities to solve problems • Rewrite rational expressions • Create equations that describe numbers or relationships • Understand solving equations as a process of reasoning & explain the reasoning • Represent & solve equations and inequalities graphically
Functions:		• Define & understand function. • Compare properties of two functions represented in different ways • Interpret linear functions & give examples of non-linear functions • Construct a function to model a linear relationship	**Functions:**	• Understand the concept of function & use function notation • Interpret functions that arise in applications in terms of the context • Analyze functions using different representations • Build a function that models a relationship between two quantities • Build new functions from existing functions • Construct & compare linear, quadratic, & exponential models		• Interpret functions that arise in applications in terms of the context • Analyze functions using different representations • Build a function that models a relationship between two quantities • Build new functions from existing functions • Construct & compare linear, quadratic, & exponential models to solve problems • Extend the domain of

Formatted by Erin Fraser, CFCC Committee Member

Integrated Pathway

MS Domains	7th Grade CC	8th Grade CC	HS Domains	Model Math 1 (Focus on linear & exponential w/ integer exponents)	Model Math 2 (Focus on quadratic)	Model Math 3
Number System:	• Arithmetic operations on rational numbers	• Know that there are irrational numbers & approximate them by rationals.	**Number & Quantity:**	• Reason quantitatively & use units to solve problems	• Extend properties of exponents to rational exponents • Use properties of rational & irrational numbers • Perform arithmetic operations with complex numbers • Use complex numbers in polynomial identities & equations	• Use complex numbers in polynomial identities & equations
Expressions & Equations (including Ratio & Proportions):	• Unit rates • Recognize & represent proportional relationships • Solve multistep ratio & percent problems • Generate equivalent expressions • Solve problems using numerical/algebraic expressions & equations	• Know and apply properties of exponents • Use square root & cube root to solve equations • Use scientific notation to estimate and compare quantities • Perform operations with numbers expressed in scientific notation • Graph proportional relationships interpreting unit rate as slope. Compare different proportional relationships represented in different ways. • Use similar triangles to explain why slope is a constant between any two distinct points on a line on the coordinate plane • Derive and use equation $y = mx + b$ • Solve linear equations in one variable • Analyze & solve linear systems	**Algebra:**	• Interpret the structure of expressions • Create equations that describe numbers or relationships • Understand solving equations as a process of reasoning & explain the reasoning • Solve equations & inequalities in one variable (including those with absolute value) • Solve systems of equations • Represent & solve equations and inequalities graphically	• Interpret the structure of expressions • Write expressions in equivalent forms to solve problems • Perform arithmetic operations on polynomials • Create equations that describe numbers or relationships • Solve equations & inequalities in one variable (including those with absolute value) • Solve systems of equations	• Interpret the structure of expressions • Write expressions in equivalent forms to solve problems • Understand the relationship between zeros & factors of polynomials • Use polynomial identities to solve problems • Rewrite rational expressions • Create equations that describe numbers or relationships • Understand solving equations as a process of reasoning & explain the reasoning • Represent & solve equations and inequalities graphically
Functions:		• Define & understand function. • Compare properties of two functions represented in different ways • Interpret linear functions & give examples of non-linear functions • Construct a function to model a linear relationship	**Functions:**	• Understand the concept of function & use function notation • Interpret functions that arise in applications in terms of the context • Analyze functions using different representations • Build a function that models a relationship between two	• Interpret functions that arise in applications in terms of the context • Analyze functions using different representations • Build a function that models a relationship between two quantities • Build new functions from existing	• Interpret functions that arise in applications in terms of the context • Analyze functions using different representations • Build a function that models a relationship between two quantities (Composition of functions & Inverse

Getting to Calculus Sooner:

Two Compacted Pathways

- **Traditional Compacted Pathway**: complete content of 7th, 8th, and HS Algebra I in grades 7 (Compacted 7th Grade) and 8 (8th Grade Algebra I) enabling them to finish Algebra II by the end of the sophomore year.
- **Integrated Compacted Pathway**: complete content of 7th, 8th, and Mathematics I in grades 7 (Compacted 7th Grade) and 8 (8th Grade Mathematics I), enabling them to complete Mathematics III by the end of the sophomore year

Both prepare students for Precalculus in their junior year and Calculus in their senior year.

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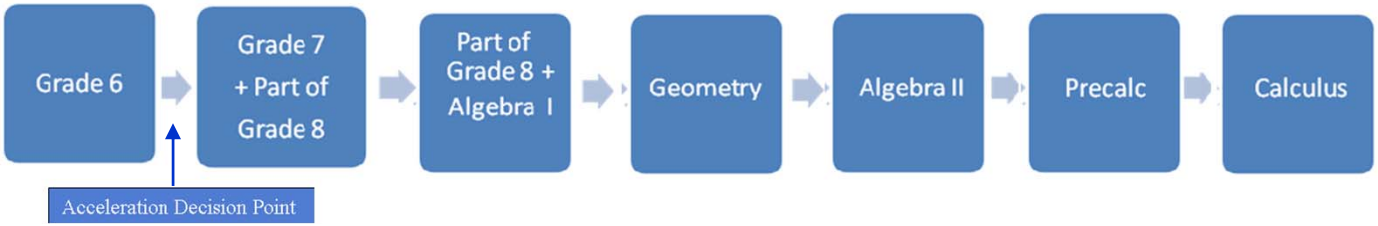
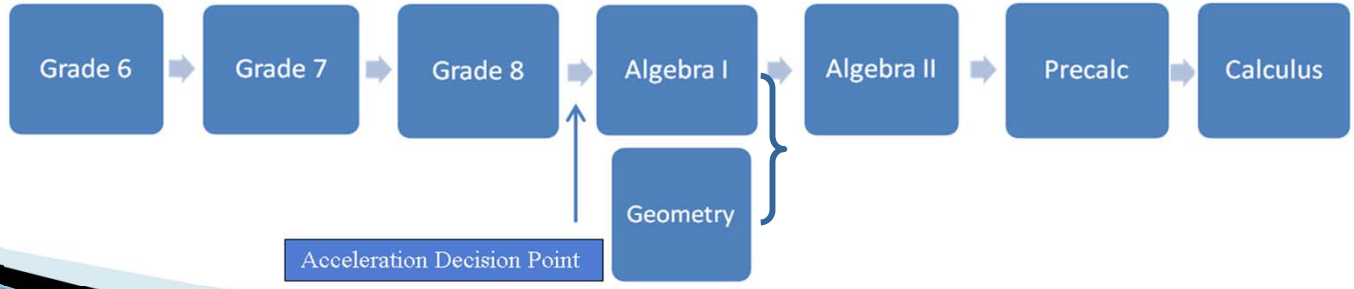
High School Traditional Pathway

- This option is a compaction of four years of grades 6,7,8 and Alg I standards into three years of instruction in middle school or doubling up of courses during the freshman year.

Model 1: Compacting in Middle School

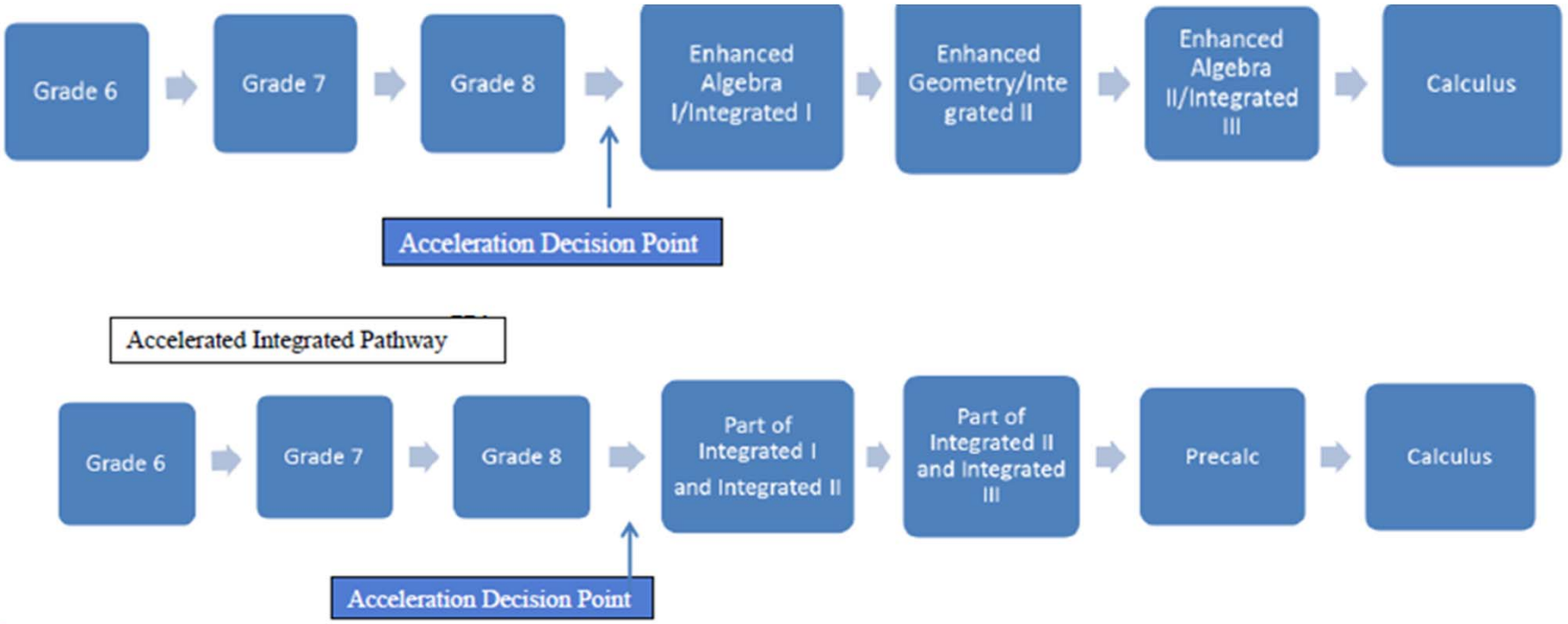


Model 2: Doubling up in High School



High School Integrated Pathway

- This option is a compaction of three years of high school standards into two years of instruction.




Critical Areas
bring FOCUS
to the New Standards

Desired Outcomes

Participants will

- Become familiar with the **Critical Areas** of either the 5th grade, 8th grade or Algebra I.

- Understand how the **Critical Areas** help organize and bring focus to the grade level standards.

Critical Areas

- There are two to four critical areas for instruction in the introduction for each grade, model course or integrated pathway.

- They bring focus to the standards at each grade by providing the big ideas that educators can use to build their curriculum and guide instruction

Investigating Focus (25 minutes)

- Participants work in teams of three to four on their grade level/course critical areas.
- Read your Critical Area and underline the key words that help summarize this area.
- On your recording sheet, indicate which standards seem to fall with your Critical Area.
- In your team have one person share the key words for their area and one interesting insight.
- As a team, discuss how Critical Areas can help organize and bring focus to the grade level standards.

Review of Objectives for this session

- ✓ Provide the overarching goal of the 2013 Draft California Mathematics Framework
- ✓ Highlight the shifts- increased focus, coherence and rigor
- ✓ Explain some of the changes between the 1997 framework and the 2013 mathematics framework
- ✓ Show the organization of the 2013 Draft California Mathematics Framework
- ✓ Point out some key points in the draft math framework
- ✓ Look at the differences between "traditional" and "integrated"
- ✓ Examine what "Math Modeling" means
- ✓ Acceleration Pathways
- ✓ Engage in a "critical area" activity on the framework